

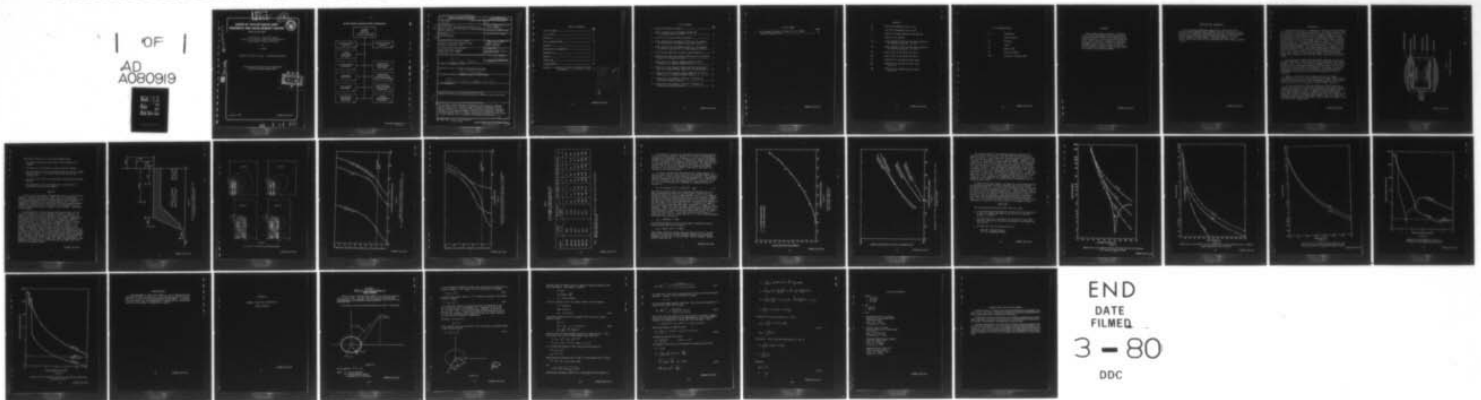
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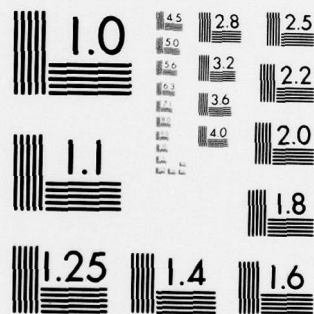
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THE EFFECT OF A SATURATED SHIELD DESIGN ON THE DESIGN, WEIGHT, --ETC(U)  
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**DAVID W. TAYLOR NAVAL SHIP  
RESEARCH AND DEVELOPMENT CENTER**

Bethesda, Maryland 20084

The Effect of a Saturated Shield  
Design on the Design, Weight and Performance of  
Superconductive Acyclic Machinery

by

G. Green

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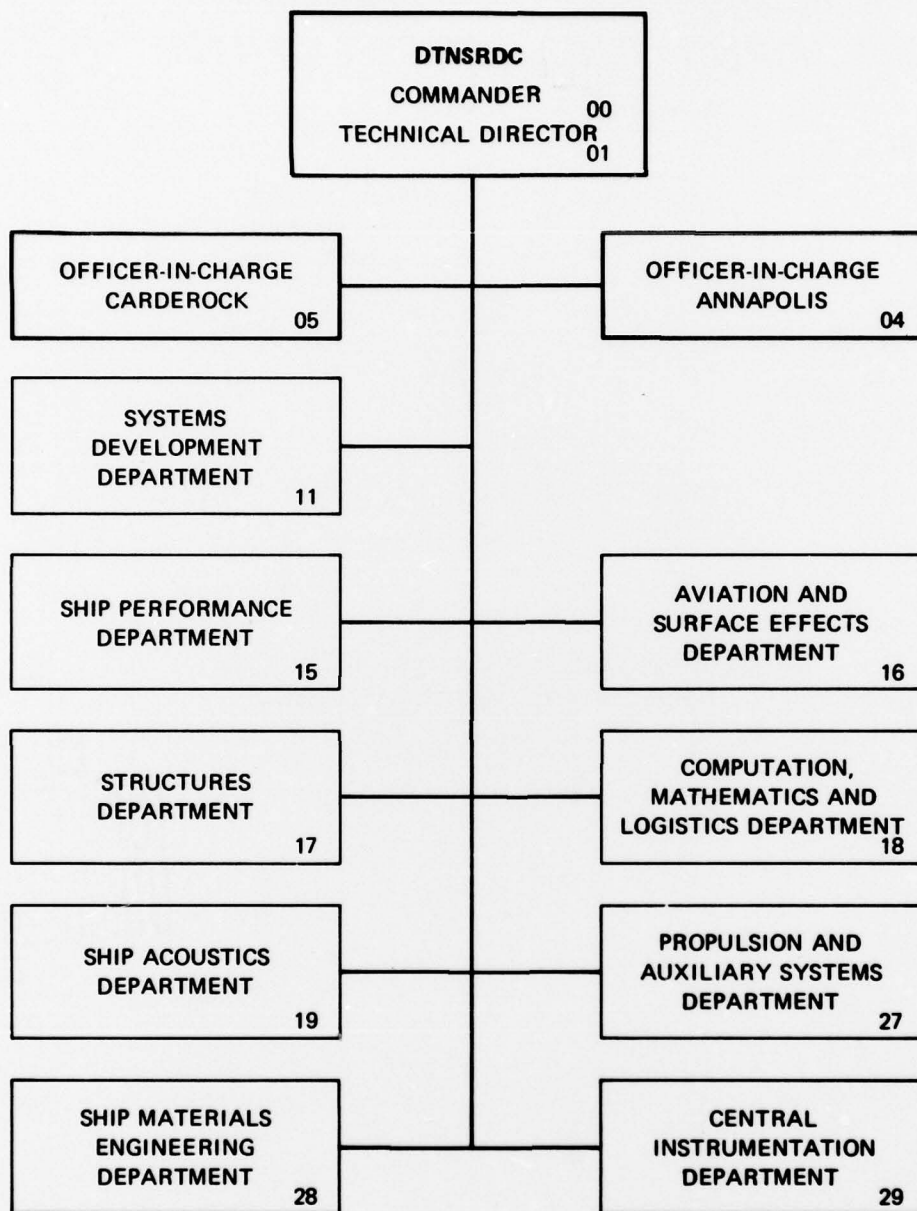
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER PAS-79-29	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER (9)
4. TITLE (and Subtitle) 6 The Effect of a Saturated Shield Design on the Design, Weight, and Performance of Superconductive Acyclic Machinery.		5. TYPE OF REPORT & PERIOD COVERED Interim Report.
7. AUTHOR(s) 10 Geoffrey Green		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS David W. Taylor Naval Ship R&D Center Annapolis, Maryland 21402		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Sea Systems Command Washington, D.C. 20362		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 63508N, 50380 SL, 16761, 2722-100
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 17 S0380SL		12. REPORT DATE 11 29 Jun 79
		13. NUMBER OF PAGES
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release: Distribution Unlimited 16 S0380SL		15. SECURITY CLASS. (of this report) 12 36
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) 14 DTNSRDC/PAS-79-29		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Magnetic Shielding, Superconducting Machinery, Machine Sizing, Stay Field		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The estimated weight advantage of designing a superconducting homopolar acyclic machine with a saturated magnetic shield is presented, and the possible power or flux loss due to the saturation of the iron is reported. A method of estimating the magnetic field at 15.23 centimeters from the saturated magnetic shield is shown in addition to indicating how the fields at large distances vary for hexapole and quadrupole designs.		

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#### NOTATION

B1	The field produced by Coil 1 at R1
B2	The field produced by Coil 2 at R2
E	Percent (%) weight reduction from SMR 97H
R	Radius of the machine
r <sub>01</sub>	Radial distance (15.24 cm from outer surface of the machine) along the of Coil 1
r <sub>02</sub>	Radial distance (15.24 cm from outer surface of the machine) along the of Coil 2
R <sub>01</sub>	Outer radius of the shield for Coil 1
R <sub>02</sub>	Outer radius of the shield for Coil 2
$\phi_e$	Effective flux through the rotor drums
$\phi_{e1}$	Effective flux through the rotor drums for coil 1
$\phi_{e2}$	Effective flux through the rotor drums for coil 2

# LIST OF ABBREVIATIONS

cm	centimeters
j	current density
kg	kilogauss
in	inches
NI	ampere turns
OD	outside diameter
TRIM	Triangular Irregular Mesh



# ABSTRACT

The estimated weight advantage of designing a superconducting homopolar acyclic machine with a saturated magnetic shield is presented, and the possible power or flux loss due to the saturation of the iron is reported. A method of estimating the magnetic field at 15.24 cm (6 in) from the saturated magnetic shield is shown in addition to indicating how the fields at large distances vary for hexapole and quadrupole designs.

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#### ADMINISTRATIVE INFORMATION

This study was accomplished under David W. Taylor Naval Ship Research and Development Center (DTNSRDC) Work Unit 2722-100 as part of the supporting technology development for the Naval Sea Systems Command (NAVSEA) Superconductive Propulsion Machinery Program, Task 16761, Project S0380-SL, Element 63508N. The Program Manager is Mr. A. Chaikin, NAVSEA 05R. This work was completed and a draft report prepared in 1976.

## INTRODUCTION

Superconductive acyclic, or homopolar, machines are being developed by the Navy for ship propulsion application. The machine contains three major elements; a superconducting coil to generate magnetic flux, an armature circuit which converts electrical power to mechanical power in the presence of a magnetic field, and a ferromagnetic shield to shape and contain the flux. A machine may include a single solenoid (dipole) magnet winding, as illustrated conceptually in Figure 1, or employ two or three solenoids (i.e. quadrupole or hexapole windings). In Figure 1, the superconducting magnet within the helium vessel, or dewar, is the innermost machine element. The intense magnetic flux generated in the bore of the solenoid is attracted by the ferromagnetic shield forcing virtually all of the magnetic flux to radially transverse the rotor twice. When current is passed through brushes and axially down the copper rotor conductors, the resulting Lorentz interaction provide motor action.

In the design of superconducting homopolar machines, the practice has been to design the magnetic shield in the unsaturated state, 15-20 kilogauss (KG). That is, to design the magnetic shield so that the flux leakage, close to the machine surface (typically 15.24 cm (6 in) from the shield), doesn't exceed a flux density of 100 gauss. The low flux leakage was established so that the magnetic field generated by the superconducting magnet would not effect the operation of near-by-equipment.

However, it was found that relaxation of the flux leakage requirement could result in a very large machine weight reduction due to the mass density and volume of the ferromagnetic shield. This study was undertaken to quantify this potential machine weight reduction.

In order to establish the weight benefit of relaxing the flux leakage requirements, an analysis was conducted and the machine weight advantage evaluated for a specific machine design and power rating (i.e. a 1.27 weber hexapole design). In addition, a scaling method was established to extend the weight reduction estimates for a specific flux leakage to machines of different power ratings. The field values for the hexapole and quadrupole machine at large distances are shown to vary as  $1/R^3$  and  $1/R^4$ , respectively.

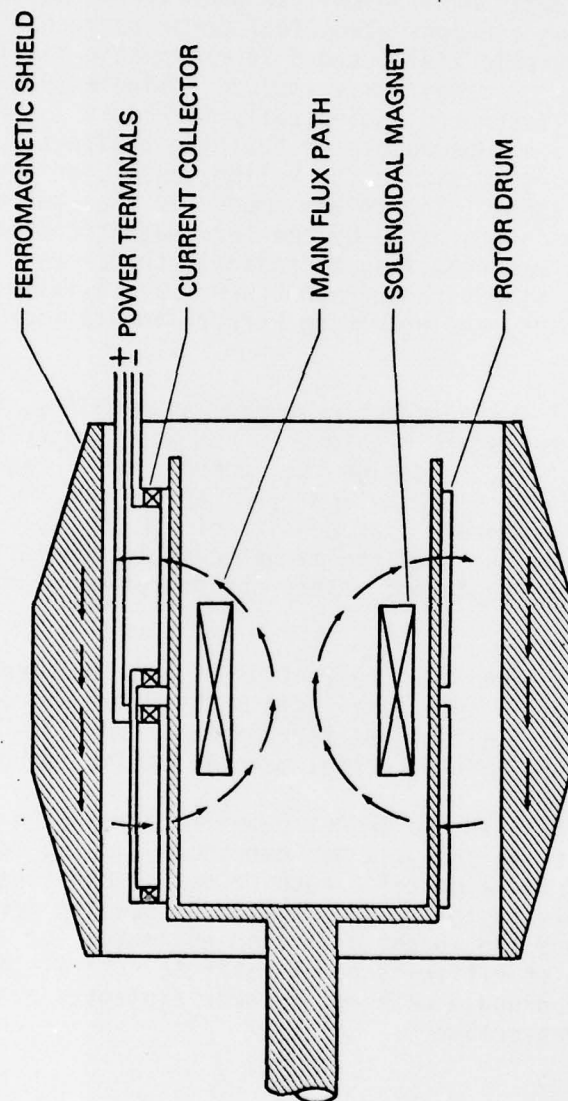


Figure 1  
Shaped Field Machine Concept

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The present analysis will cover the following areas:

- o The weight advantage available due to the allowable flux leakage.
- o An evaluation of the power loss due to the flux leakage.
- o An evaluation of a linear scaling equation for the flux leakage near the machine (i.e. 15.24 cm (6 in) from the outside diameter, OD).
- o The scaling of fields at large distances from different machine designs.
- o The comparison of the flux leakage for a shielded and an unshielded (self-shielded) machine.

#### ANALYSIS

In this study, two computer programs were used for calculating the magnetic fields of these machines. The first is a 2-dimensional triangular irregular mesh (TRIM) computer program with the capabilities of calculating magnetic fields in the presence of iron. TRIM was developed at Argonne National Laboratory. The second computer program (IMAGE) uses the Biot-Savart Law to calculate the magnetic field at specified "mesh points" due to a number of "coil regions", and assumes no iron regions.

The configuration of the hexapole machine design used, and the region of interest for the flux leakage, is shown in Figure 2. An average flux density of 15 KG in the shield of a hexapole design was used as the base case (TRIM run SMR 97B). Using TRIM, a percentage of the magnetic shield was removed and the flux leakage was noted at several locations. This information is presented in Table 1 and the flux plots for several of these cases are shown in Figures 3. The graphical representation of this data is shown on Figures 4 and 5, showing the potential weight reduction due to the increase in flux leakage. In addition, Figures 4 and 5 indicate that designing a shield at saturation (20 KG) provides a 20% decrease in the shield weight with very little flux leakage. This becomes significant, when it is realized that the weight of the shield in the larger machines, 40,000 horsepower (HP), contribute about 50-80% of the total weight of the machines. In Figure 4, field values at the larger radii were calculated based on the assumption that they varied as  $1/R^3$ , which will be shown later to be correct for a hexapole design.

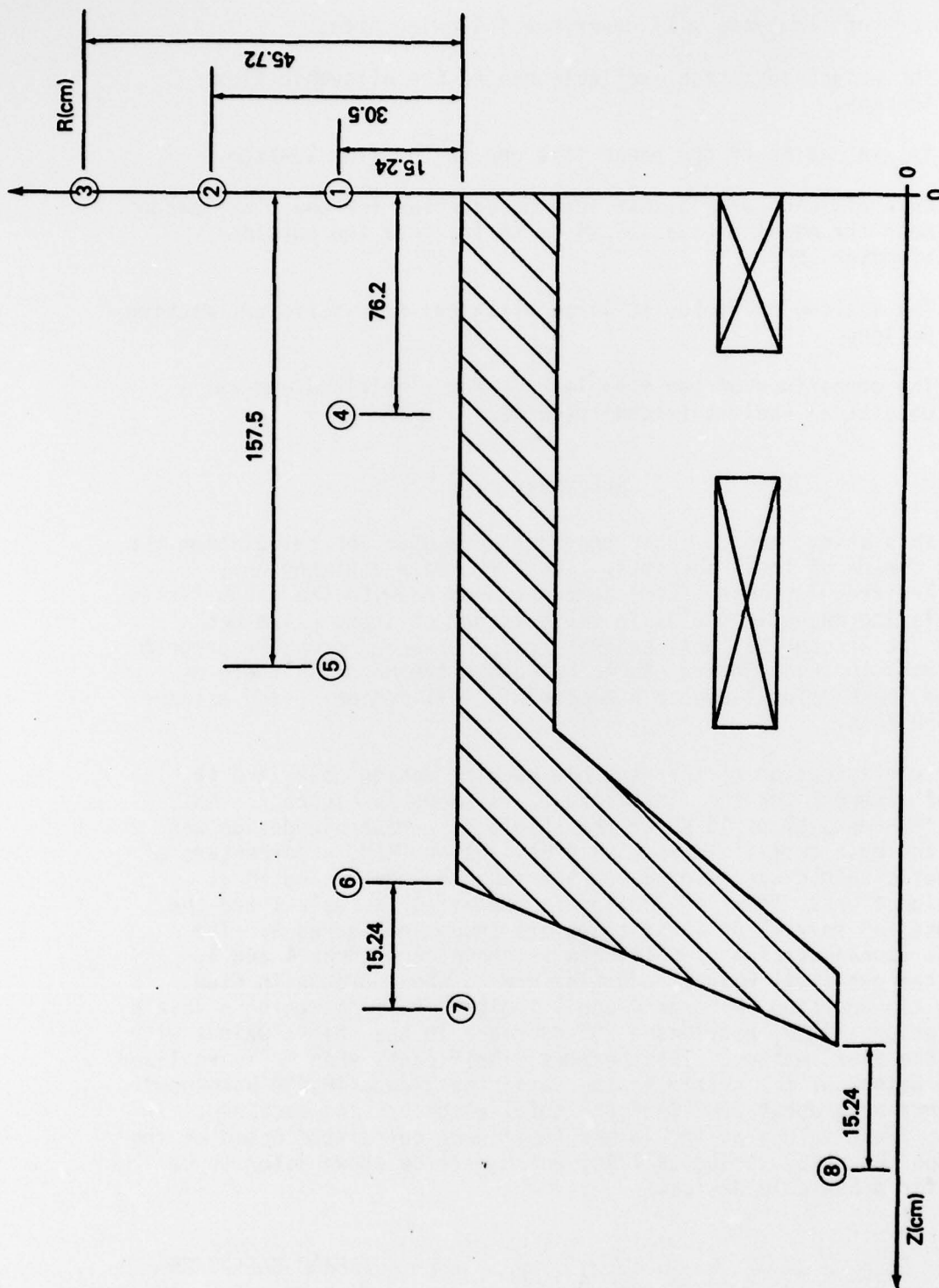


Figure 2  
Basic Configuration of Magnet Systems and  
Flux Leakage Evaluation Points



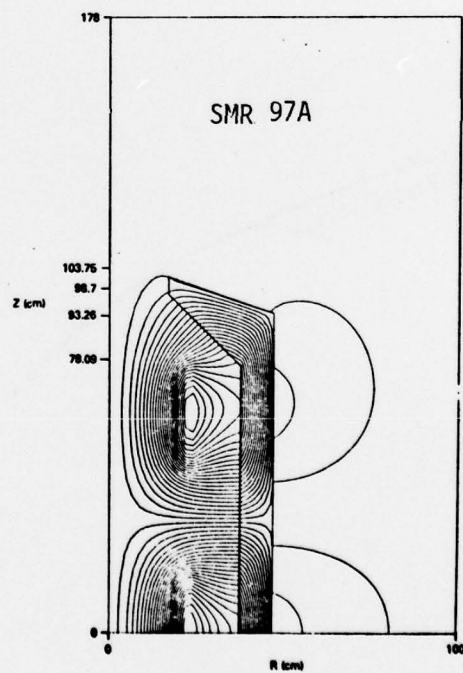
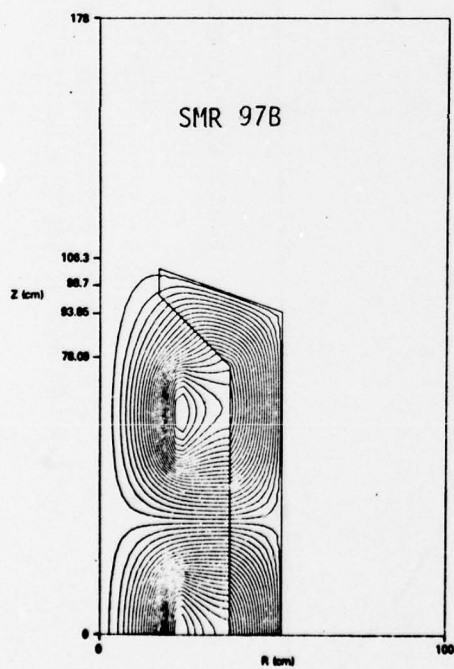
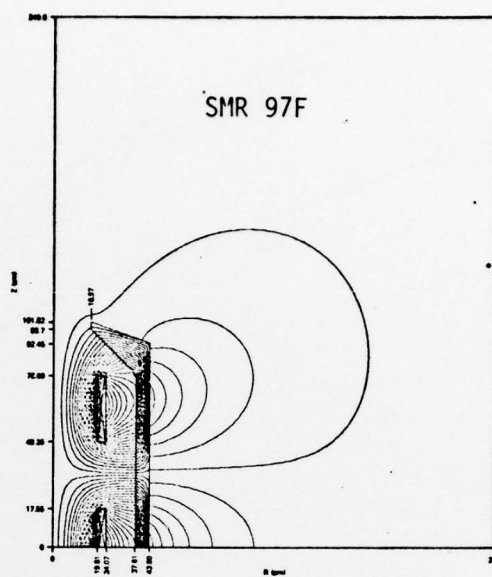
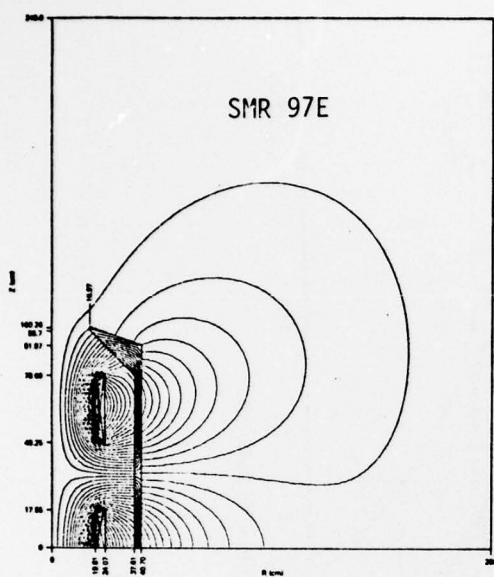


Figure 3

Flux Plots for Various Shielding Designs

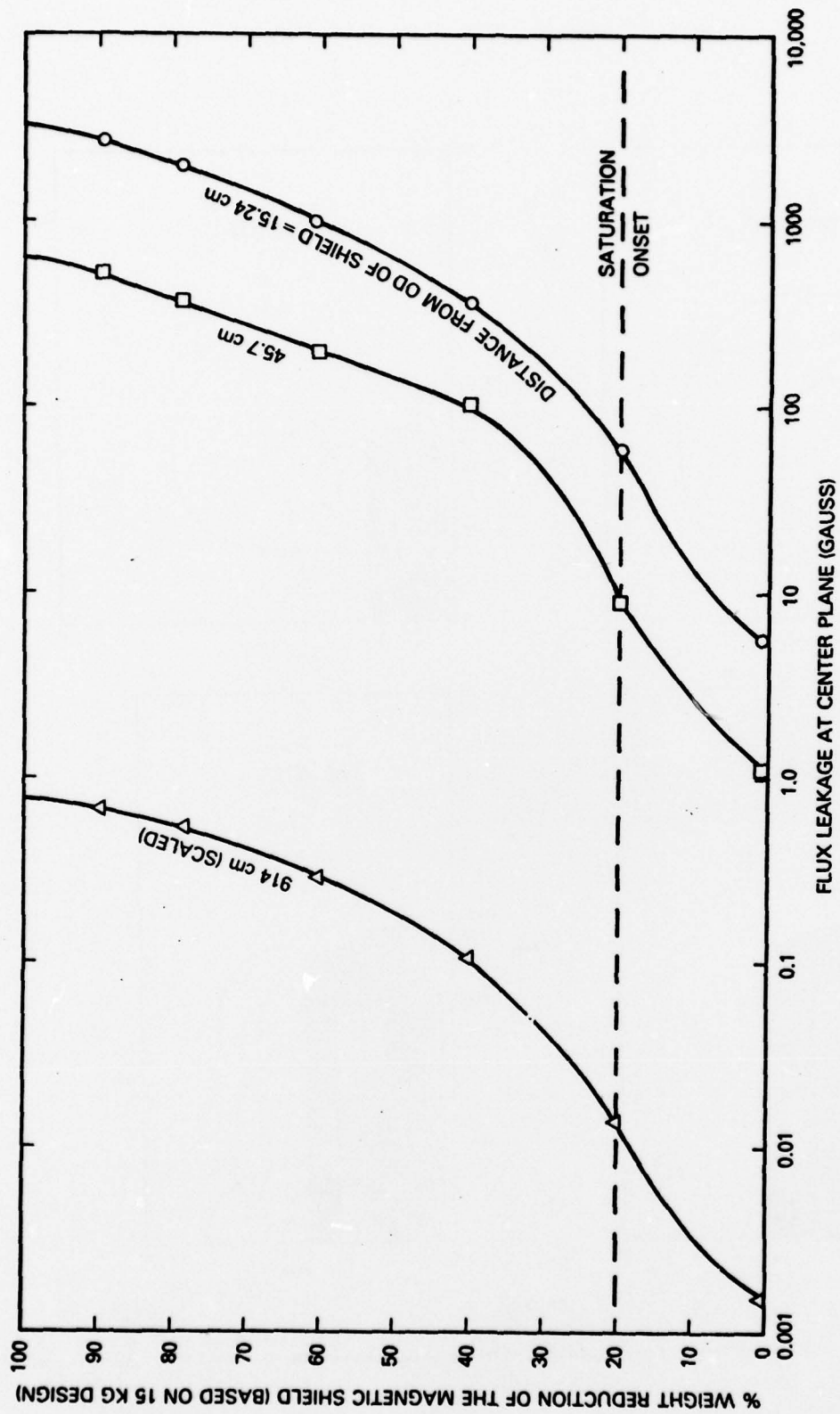


Figure 4  
Weight Reduction of the Magnetic Shield Vs.  
Flux Leakage at Various Radial Distances on the Center Plane

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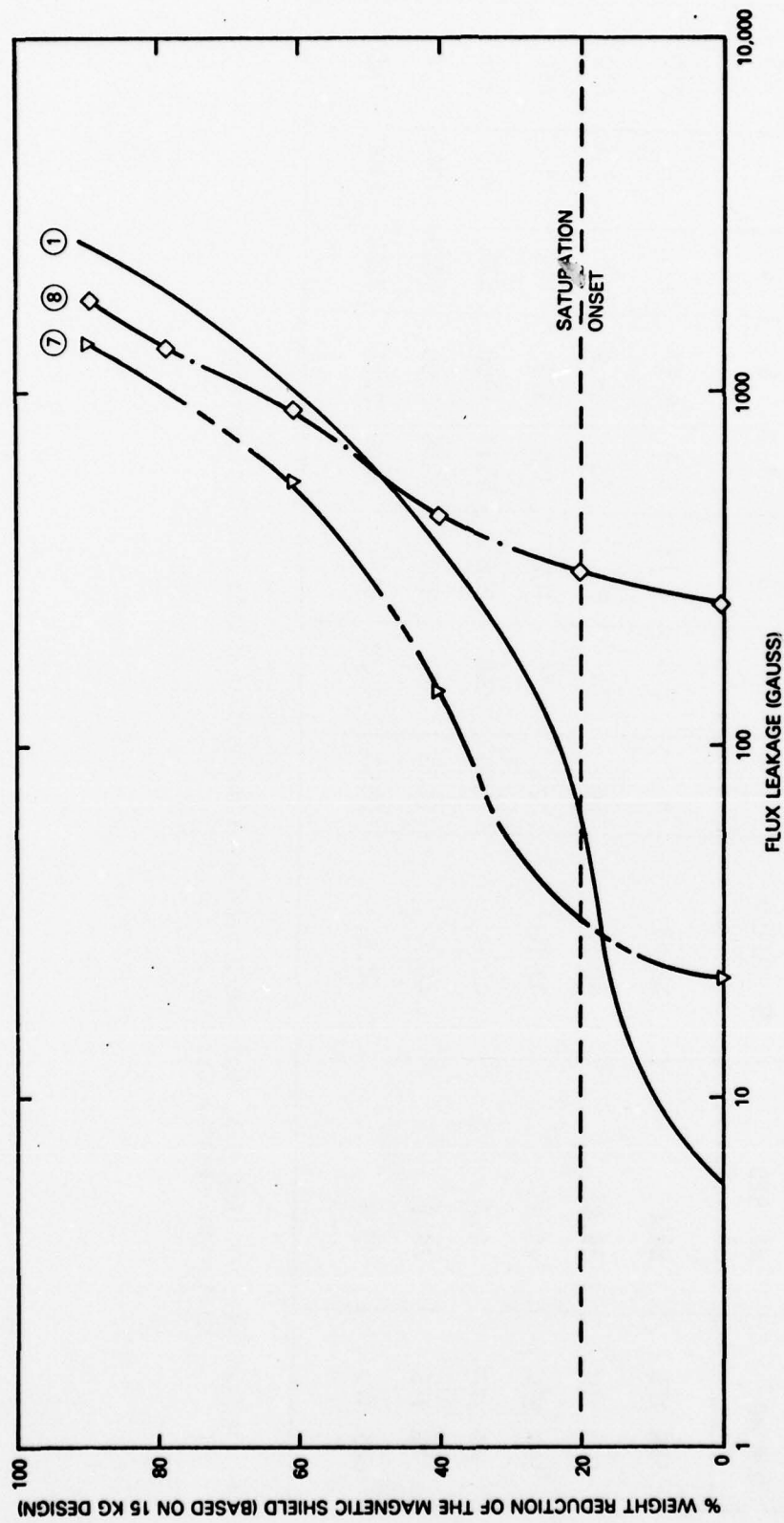


Figure 5  
Weight Reduction of the Magnetic Shield Vs. Flux Leakage  
at Various Locations for 1.27 Weber Hexapole Design

TABLE 1

FLUX LEAKAGE OF VARIOUS TRIM RUNS FOR  
A 1.27 WEBER HEXAPOLE MAGNET DESIGN

RUN NO.	% SHIELD WT. RED	OUTER RADIUS OF SHIELD (cm)	FLUX LEAKAGE (GAUSS) AT LOCATION (See Fig 2)							
			1	2	3	4	5	6	7	8
SMR 97B	BASE	52.63	5.7	2.0	1.13	6.4	9.17	11.9	22	250
SMR 97H	19.87	49.98	60	23	9	56	58.9	29	31.8	310
SMR 97A	40.2	47.03	374	175	107	335	370	195	143	450
SMR 97F	60.7	43.8	1050	458	205	1114	1000	537	562	900
SMR 97E	78.5	40.7	2000	860	383	2300	1900	1060	1042	1360
SMR 97D	89.5	38.61	2650	1080	519	3000	2225	1260	1400	1850

NOTE: SMR 97A, SMR 97B, etc. were used as a numbering system for designating  
different machine designs



The removal of a portion of the magnetic shield has been shown to have quite a weight benefit. However, it also has a disadvantage of increasing the reluctance of the magnetic circuit with the consequent decrease flux and machine power, as indicated in Figure 5 and Figure 6. In fact, the removal of the magnetic shield reduces the effective flux by approximately 25% at constant field excitation. This decrease in power or flux can be easily compensated by increasing the magnet excitation (i.e. amp-turns) slightly, resulting in less than a 2% increase in the machine diameter.

In order to evaluate the approximated flux leakage density at a specific radial distance (15.24 cm (6 in) from outside diameter, OD, of the machine), a linear scaling technique was structured. The technique requires knowledge of the field, flux and geometry of a reference coil (coil 1) to estimate the field characteristic of a second coil (coil 2). This scaling equation for a hexapole or quadupole machine is defined as:

$$B_2 = B_1 (\phi_{e2}/\phi_{e1}) \{[r_1^2 - r_{01}^2]/[r_2^2 - r_{02}^2]\} \quad (1)$$

The scaling equation used in this study was derived based on a 1.27 weber hexapole design (SMR 97H) and then used to predict the flux leakage at higher flux level hexapole machines (i.e. up to 5.0 webers) and the predictions of hexapole machines with varying amount of magnetic shielding. Since the scaling equation used in this study was linear, the non-linear effects become apparent when the results were compared to the TRIM output. This non-linear effect is illustrated in Figure 7. The linear scaling technique developed, in equation 1, appear to over predict at higher flux levels and under prediction for reductions in the shielding. A more accurate prediction could be made by adding to the scaling equation, a function that would empirically account for some of these non-linear effects. The linear correction factor ( $F_{c2}$ ) for scaling up (i.e. coil 2 is larger than coil 1) in flux from the hexapole design SMR 97H is defined as:

$$F_{c1} = -.05404 \phi_e + 1.0669$$

The correction factor ( $F_{c2}$ ) for scaling down in shielding from the hexapole design SMR 97H is defined as:

$$F_{c2} = 7.5198 \times 10^{-3} E + .70498$$

These linear correction factors were then factored into the scaling equation and using the 5.0 weber hexapole design, the flux density leakage at 15.24 cm (6 in) was estimated. Results showed a maximum error of less than 9%.

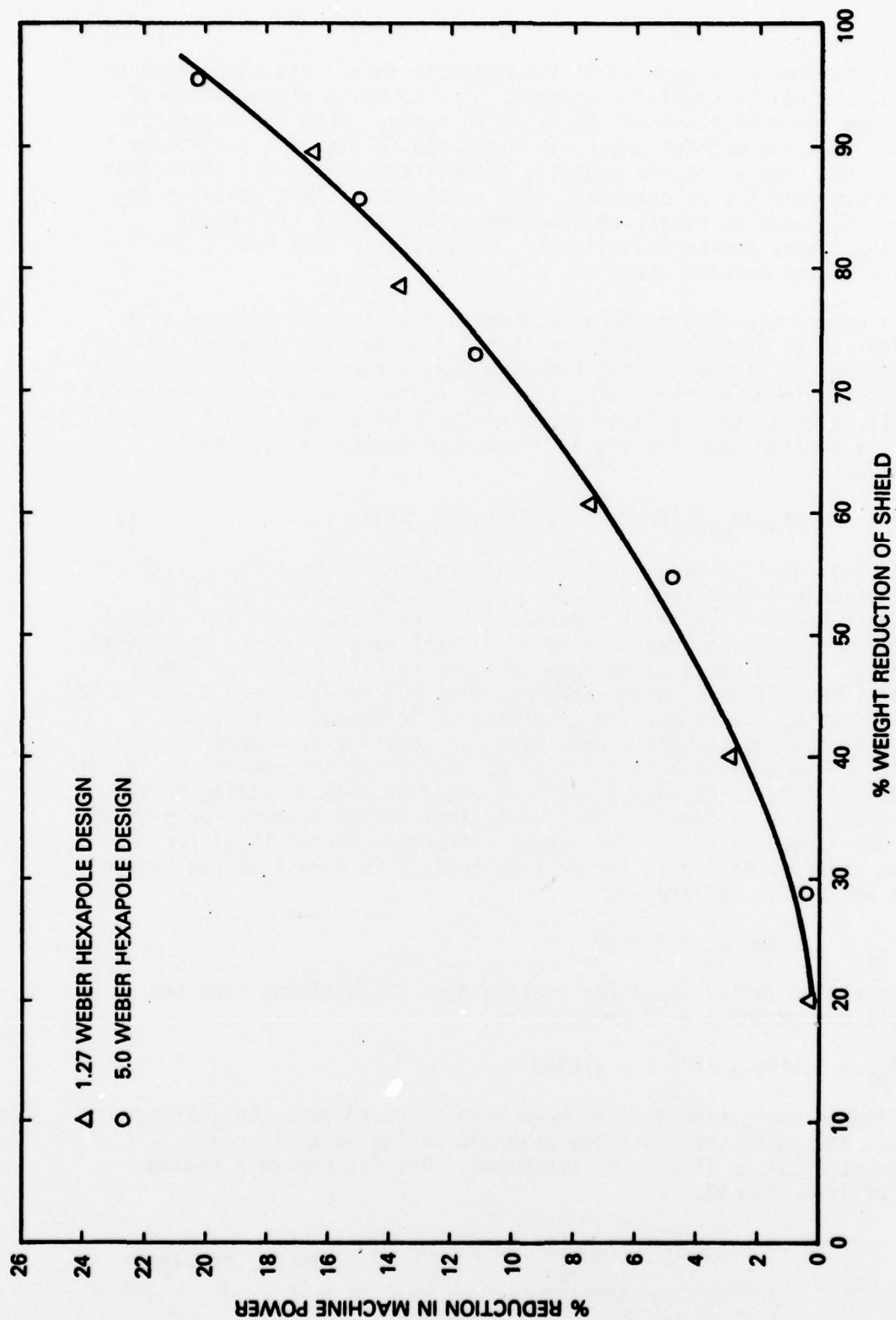


Figure 6

Shield Weight Reduction Vs.  
Machine Power Reduction



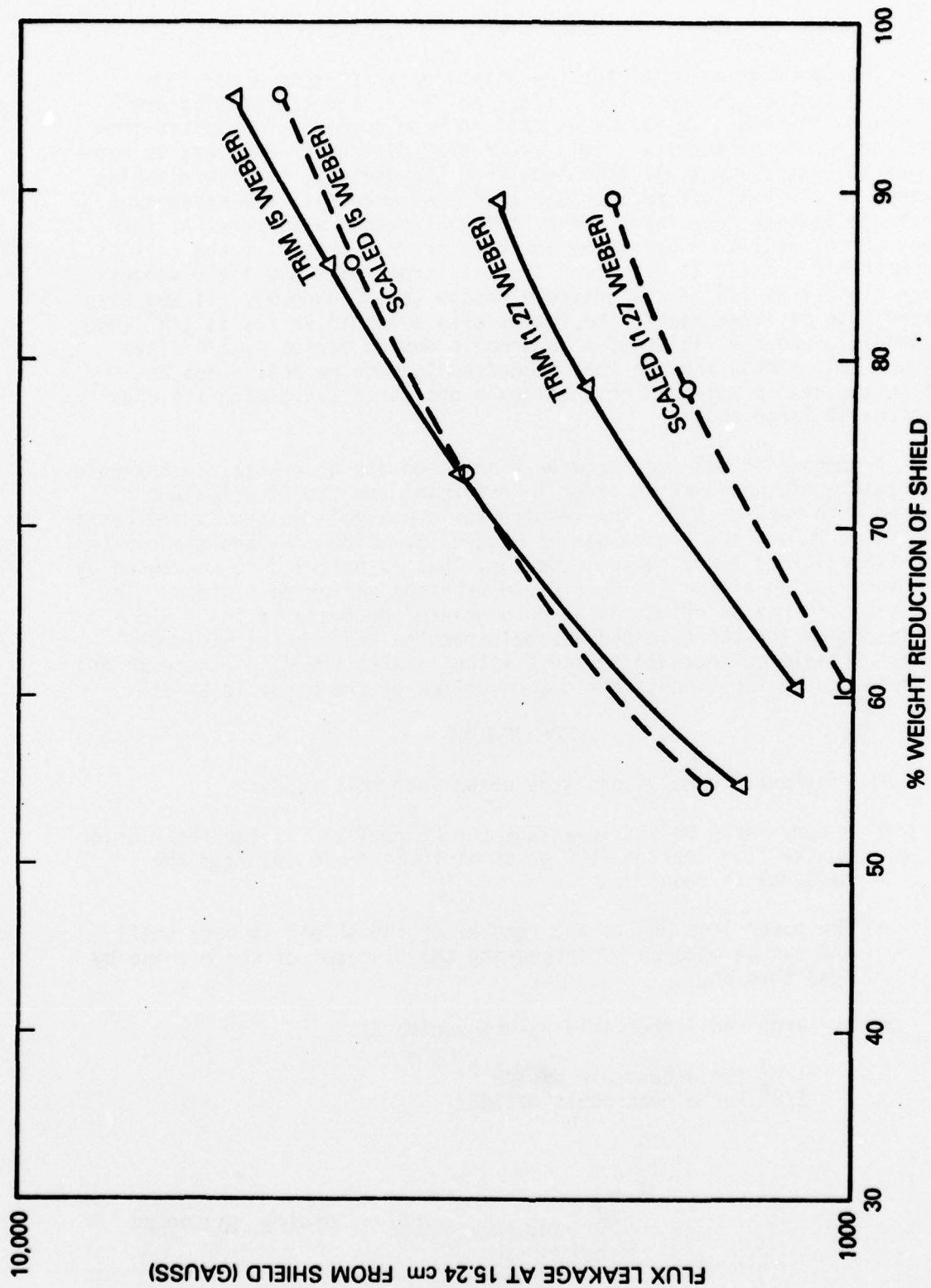


Figure 7

Comparison of the Linear Scaling Technique and the Computer Program TRIM  
(Hexapole Design)

The computer solution for the hexapole magnet system was the compared to the functions  $1/R^3$ ,  $1/R^4$ , and  $1/R^5$ , and the results are shown on Figure 8. It was found that no good comparison resulted from the curves of the magnetic field vs. radial distance. However, it was found that at these small distances from the machine, the field values seemed to all fall-off faster than  $1/R^3$ . As a result, the assumption that the leakage flux for a hexapole magnet design will vary, at the very worst, as  $1/R^3$ . Using the computer program IMAGE for the case of no magnetic shield, it was found that at large radii the field varies very closely as  $1/R^3$  for a hexapole design (see Figure 9). It was also noted that at large radii, the fields of a solenoid varies as  $1/R^3$  (see appendix), and the fields of a quadrupole design varies as  $1/R^4$  (see Figure 10). Thus, showing that a quadrupole machine design has an advantage over a solenoid or a hexapole design in decreasing its flux leakage at large radii.

A comparison was made between a shielded and an unshielded hexapole and quadrupole machines in order to determine how the flux leakage varied with machine O.D. The results for a hexapole design is indicated on Figure 11 and the quadrupole on Figure 12. Figure 11 shows a dip in the field values for a hexapole design, this is believed to be caused by the interaction of the end coil field with the center coil field. In addition, it may be noted that a much greater decrease in the field value occurs for the shielded hexapole machine than the unshielded case. This large decrease in field value is also noted in Figure 12 and represents the location of the outer surface of the magnetic shield.

#### CONCLUSIONS

The following conclusions were drawn from this study:

- o A very large weight advantage can be realized if the restriction on the flux leakage (100 gauss at 15.24 cm (6 in) from the machine) is reduced.
- o The power loss due to the removal of the shield is very small and can be made up by increasing the diameter of the machine by less than 2%.
- o At large radii the field values varies as:
  - $1/R^3$  for a hexapole design
  - $1/R^4$  for a quadrupole design

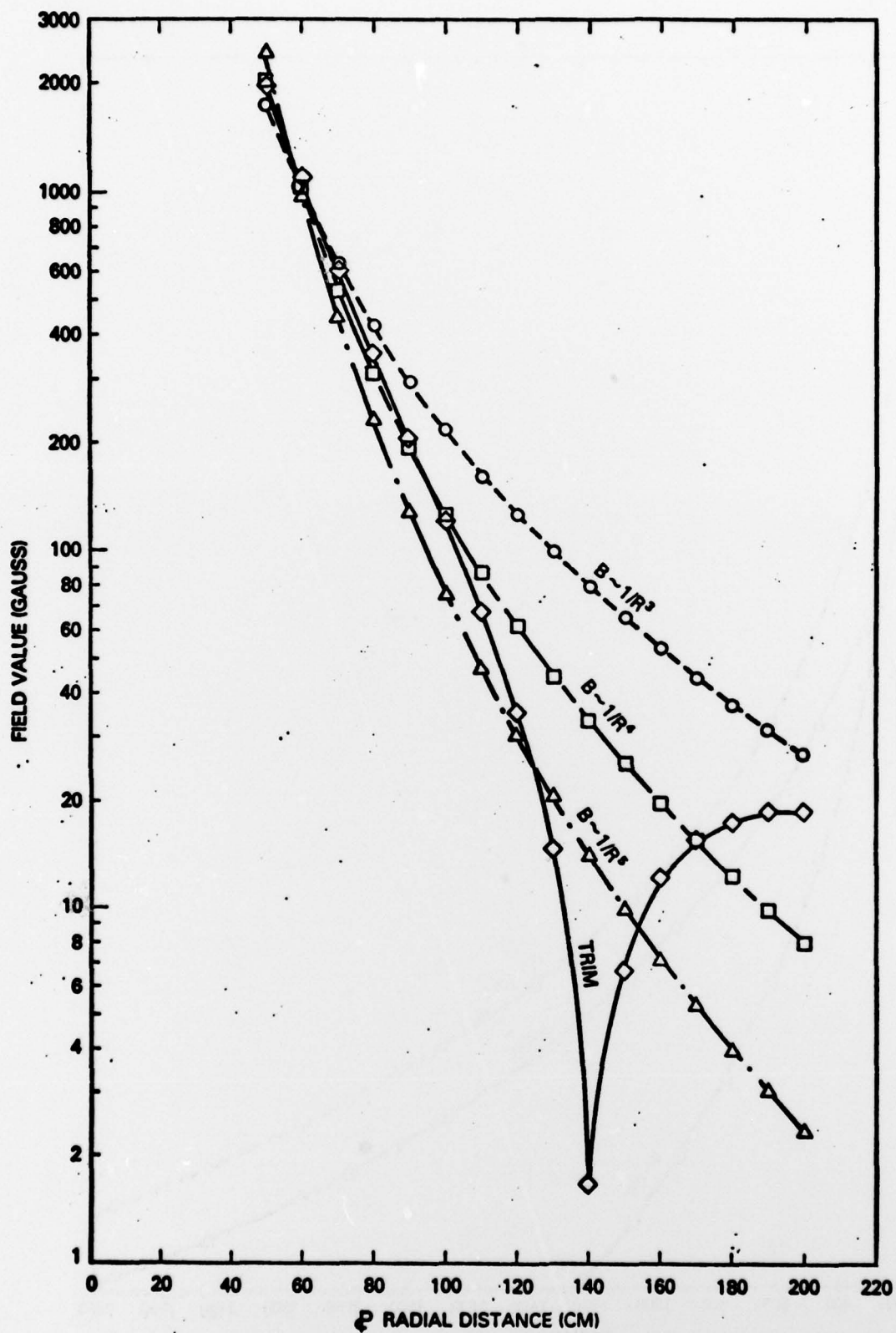


Figure 8

Comparison of the Computer Program TRIM and the Scaling Technique  
for a Hexapole Magnet System

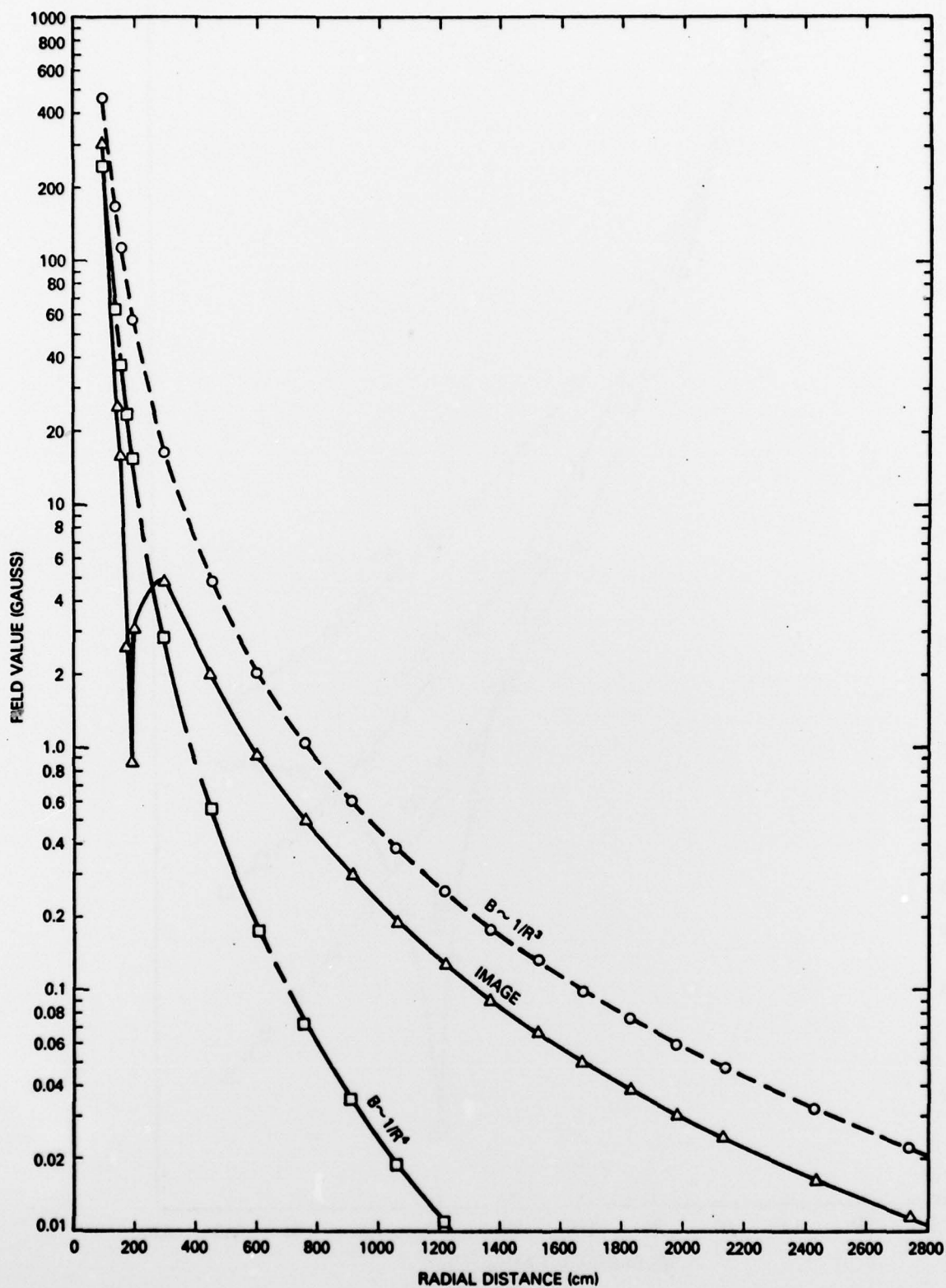


Figure 9  
Comparison of the Computer Program IMAGE and the Scaling Technique for a Hexapole Magnet System (No Iron Shielding)



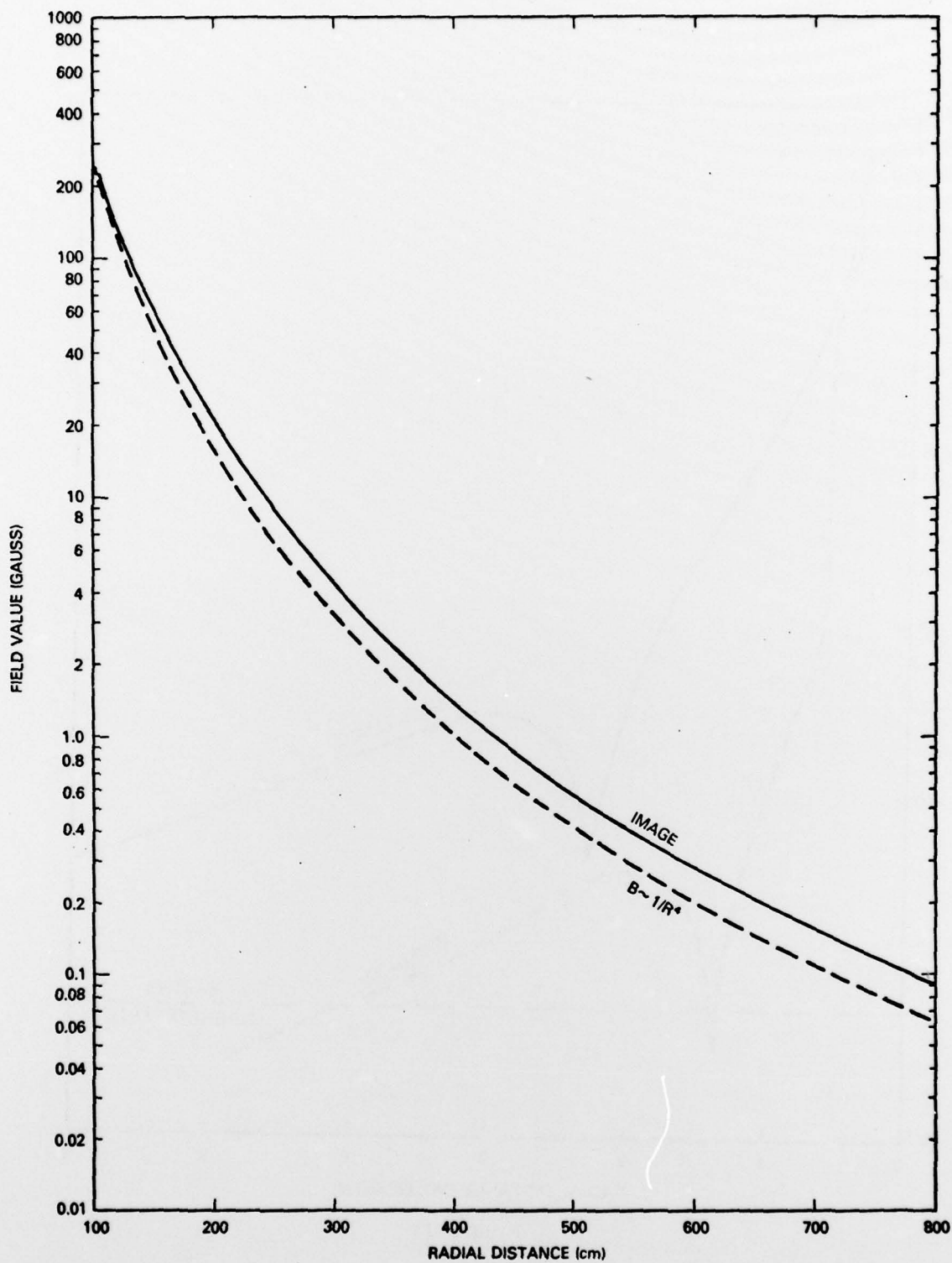


Figure 10

Comparison of the Computer Program IMAGE and the  
Scaling Technique for a Quadrupole Magnet System

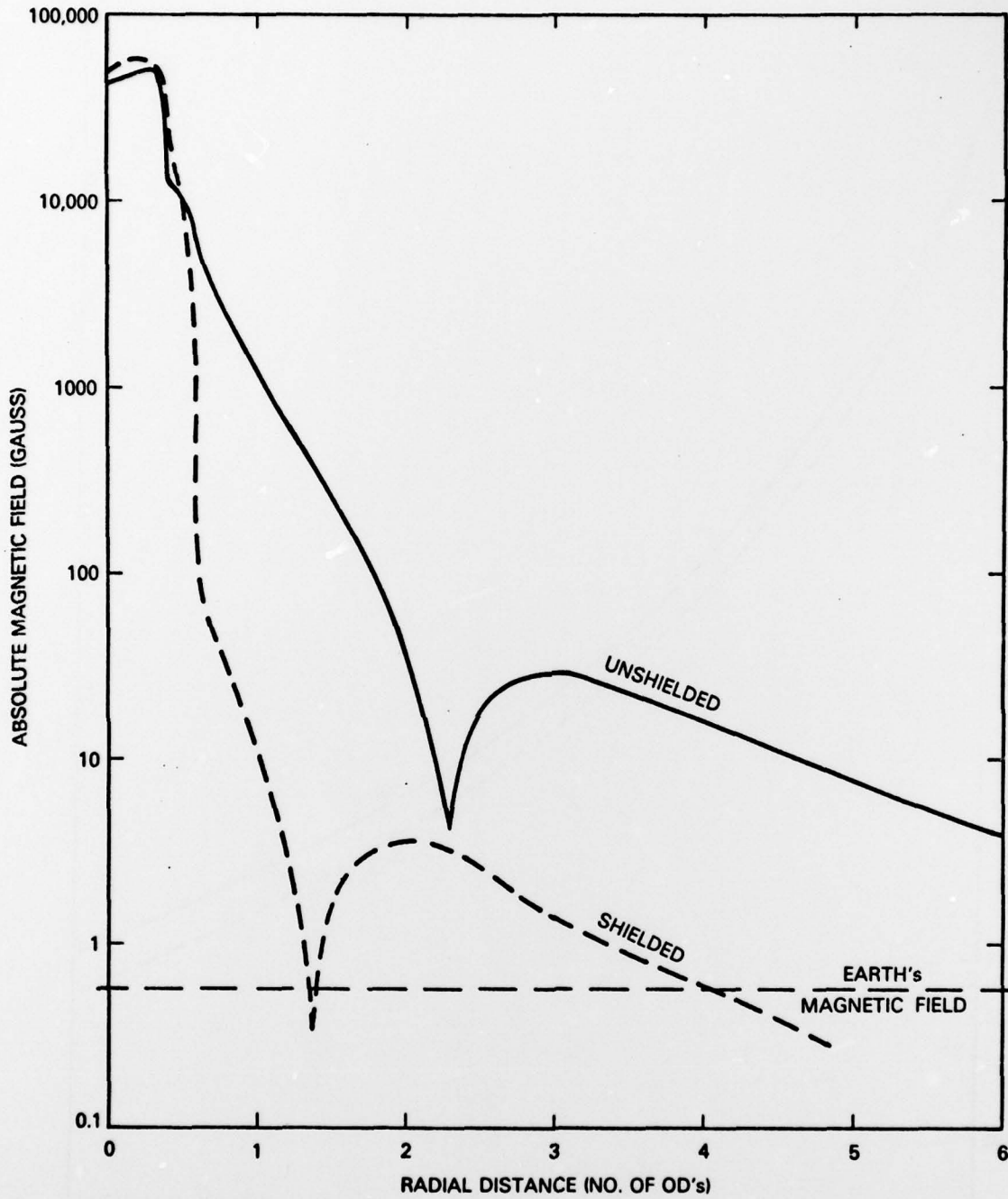


Figure 11

Comparison of the Magnetic Fields of a  
Shielded and Unshielded Hexapole Machine

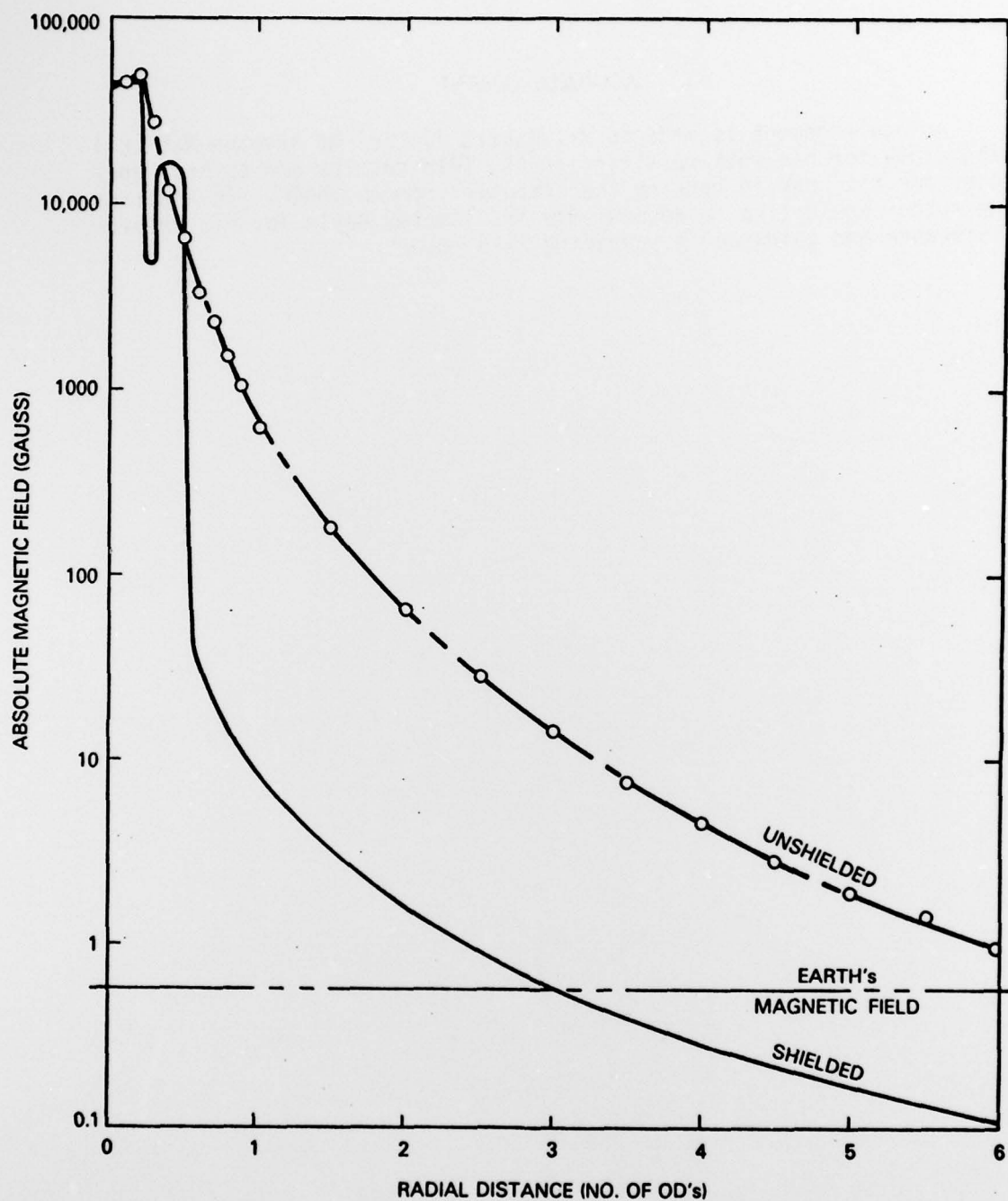


Figure 12

Comparison of the Magnetic Fields of a Shielded and Unshielded  
Quadrupole Machine

#### ACKNOWLEDGEMENT

Acknowledgement is made to Mr. Robert J. Lari of Argonne National Laboratory for his work resulting in the TRIM results and to Mr. Henry Robey for his work in running the computer program IMAGE. In addition, the author would like to acknowledge Mr. Timothy Doyle for his helpful assistance and guidance in preparing this report.



APPENDIX A

MAGNETIC FIELDS FOR A SOLENOID, AT  
LARGE DISTANCES

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A-1

APPENDIX A  
MAGNETIC FIELDS FOR A SOLENOID, AT  
LARGE DISTANCES

This derivation indicates that fields from a solenoid having no iron varies as  $1/R^3$ . The basic derivation was obtained from The Introduction to Electromagnetic Fields and Waves, by Charles H. Holt, pages 356-358.

The solenoid and the coordinate systems are shown in Figure A-1:

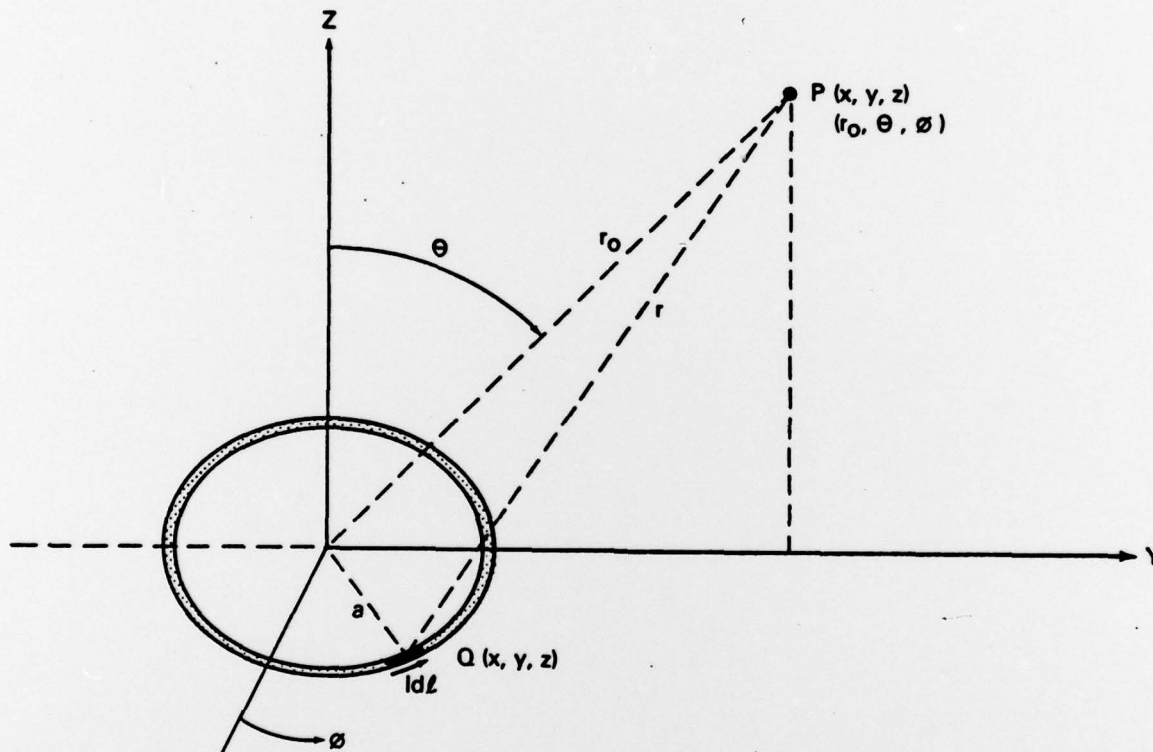


Figure A-1

Basic Equation:  $\nabla^2 \vec{A} = -\mu \vec{J}$

where  $\vec{A}$  = vector potential  
 $\vec{J}$  = current density vector  
 $\mu$  = permeability of the media

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In the RECTANGULAR COORDINATE SYSTEM, this equation has the same form as Poisson's equation. The integral solution of Equation A-1 becomes:

$$\vec{A} = \frac{\mu}{4\pi} \int_V \frac{\vec{J}}{r} dv \quad (A-2)$$

In terms of the steady current  $I$  of a filamentary conductor, the vector potential becomes:

$$\vec{A} = \frac{\mu}{4\pi} \int_C \frac{I}{r} d\vec{\ell} \quad (A-3)$$

It is noted from symmetry considerations that the magnitude of the vector potential ( $A$ ) is independent of  $\theta$ . Therefore,  $\theta$  can be set at  $\pi/2$  resulting in only a  $X$ -directional component of the vector potential ( $\vec{A} = -A_x$ ). In addition, because of the orientation of the coil and the coordinate system (spherical) used, the vector potential has only the single spherical component  $A_\theta$ .

Therefore, at the Point P,

$$A_\theta = -A_x$$

Thus, Equation (A-3) can be written in the rectangular coordinate system in the following manner:

$$A_x = \frac{\mu}{4\pi} \int_C \frac{I}{r} dx \quad (A-4)$$

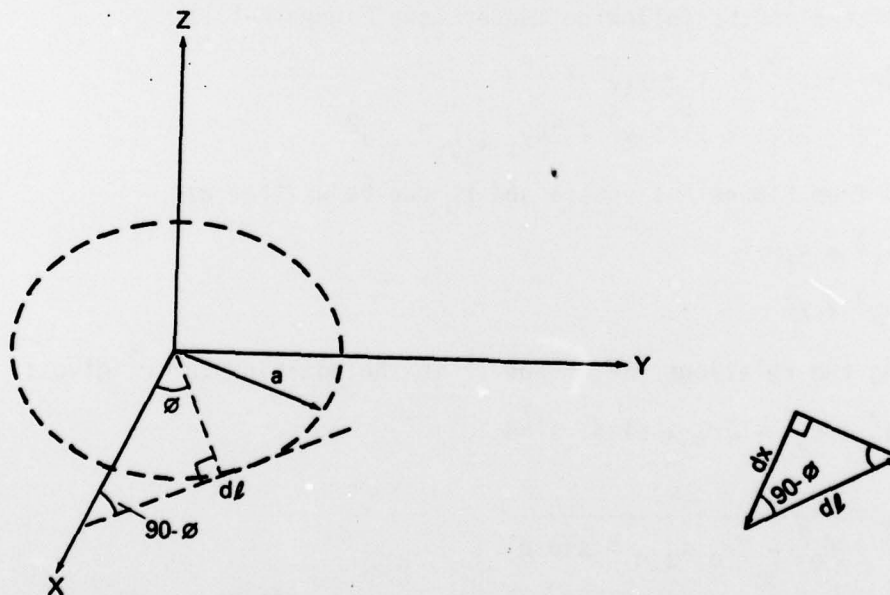


Figure A-2

Equation (A-4) can then be put in a spherical coordinate system in the following manner: (See Figure A-2 above)

$$d\ell \simeq a d\phi$$

$$\cos (90-\phi) = \frac{-dx}{d\ell}$$

$$d\ell = -dx/\cos (90-\phi)$$

Using the identity  $\sin \phi = \cos (90-\phi)$  results in the following:

$$d\ell = -dx/\sin \phi$$

$$a d\phi = dx/\sin \phi$$

$$dx = -a \sin \phi d\phi \quad (A-5)$$

Substituting equation (A-5) into equation (A-4) to give the vector potential gives:

$$A\phi = A_x$$

$$A\phi = -\frac{\mu}{4\pi} \int \frac{I}{r} (-a \sin \phi) d\phi \quad (A-6)$$

$$A\phi = \frac{\mu I a}{4\pi} \int_0^{2\pi} \frac{\sin \phi d\phi}{r}$$

Equation (A-6) has the variable (r) which is a function of  $\phi$ . Thus, r can be rewritten in the following manner (see Figure A-1):

$$r^2 = (x_1 - x)^2 + (y - y_1)^2 + z^2$$

$$r^2 = x_1^2 - 2x_1x + x^2 + y^2 - 2yy_1 + y_1^2 + z^2$$

It is noted from Figure A-1 that a and  $r_0$  can be written as:

$$a^2 = x_1^2 + y_1^2$$

$$r_0^2 = y^2 + z^2$$

Substituting the relations for  $a^2$  and  $r^2$  in the equation for  $r^2$  gives:

$$r^2 = a^2 + r_0^2 - 2r_0 a \sin \theta \sin \phi$$

and

$$r = \sqrt{a^2 + r_0^2 - 2r_0 a \sin \theta \sin \phi}$$

substituting the above relation for r into equation (A-6) results in



$$A_{\phi} = \frac{\mu I a}{4\pi} \int_0^{2\pi} \frac{\sin \phi \, d\phi}{\sqrt{a^2 + r_0^2 - 2r_0 a \sin \theta \sin \phi}} \quad (A-7)$$

This equation is very hard to solve because of the use of the elliptical integrals. However, if the following is assumed

$$a \ll r_0$$

the solution becomes greatly simplified. Then using this assumption in equation (A-7) gives the following

$$A_{\phi} = \frac{\mu I a}{4\pi} \int_0^{2\pi} \frac{\sin \phi \, d\phi}{\sqrt{1 - (2a/r_0) \sin \theta \sin \phi}} \quad (A-8)$$

Then noting that the second term in the radical is very small compared with unity, the denominator can be approximated by using a binomial expansion and neglecting all except the first two terms. Thus,

$$(1 - (2a/r_0) \sin \theta \sin \phi)^{-1/2} \approx 1 + \frac{a}{r_0} \sin \theta \sin \phi$$

When substituted into equation (A-8)

$$A_{\phi} = \frac{\mu I a}{4\pi r_0} \int_0^{2\pi} (\sin \phi) \left(1 + \frac{a}{r_0} \sin \theta \sin \phi\right) d\phi \quad (A-9)$$

Integrating equation (A-9) gives

$$A_{\phi} = \frac{\mu I S \sin \theta}{4\pi r^2} \quad \text{where } S = \pi a^2$$

The magnetic field (B) can be evaluated by taking the curl of  $A_{\phi}$

$$\mathbf{B} = \nabla \times \mathbf{A}_{\phi}$$

$$\begin{aligned} \mathbf{B} = & \frac{\vec{a}_r}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (A_{\phi} \sin \theta) - \frac{\partial A_{\theta}}{\partial \phi} \right] \\ & + \frac{\vec{a}_{\theta}}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_{\phi}) \right] \\ & + \frac{\vec{a}_{\phi}}{r} \left[ \frac{\partial}{\partial r} (r A_{\theta}) - \frac{\partial A_r}{\partial \theta} \right] \end{aligned} \quad (A-10)$$

$$B = \frac{a_r}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (A \sin \theta) \right] - \frac{a_\theta}{r} \left[ \frac{\partial}{\partial r} (rA) \right]$$

$$B = \frac{a_r}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} \left( \frac{\mu IS \sin^2 \theta}{4\pi r^2} \right) \right] - \frac{a_\theta}{r} \left[ \frac{\partial}{\partial r} \left( \frac{\mu IS \sin \theta}{4\pi r} \right) \right]$$

$$B = \frac{a_r}{r \sin \theta} \left[ \frac{\mu IS}{4\pi r^2} 2 \sin \theta \cos \theta \right] - \frac{a_\theta}{r} \left[ \frac{\mu IS \sin \theta}{4\pi} \left( -\frac{1}{r^2} \right) \right]$$

$$B = \frac{\mu IS}{4\pi r^3} [ 2 \cos \theta \vec{a}_r + \sin \theta \vec{a}_\theta ]$$

The Magnetic field along the axis  $\theta = 0^\circ$  is

$$B_{\text{axis}} = \frac{\mu IS}{4\pi r^3} [ 2 (1) \vec{a}_r + 0 \vec{a}_\theta ]$$

(A-11)

$$B_{\text{axis}} = \frac{\mu IS}{2\pi r^3} \vec{a}_r$$

The Magnetic field along the center-plane ( $\theta = 90^\circ$ ) is

$$B = \frac{\mu IS}{4\pi r^3} [ 2(0) \vec{a}_r + (1) \vec{a}_\theta ]$$

$$B = \frac{\mu IS}{4\pi r^3} \vec{a}_\theta$$

Therefore:

$$B_{\text{axis}} \sim \frac{1}{r^3}$$

$$B \sim \frac{1}{r^3}$$

(A-12)

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